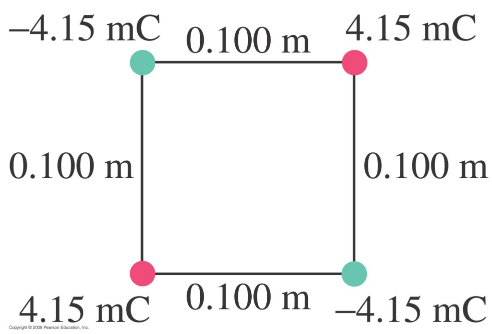

Figure 1: Problem 2

PHYS 272 (Spring 2019): Introductory Physics: Fields

Problem-solving sessions

(1). **Force between two point charges**: Two small nonconducting spheres have a *total* charge of 90μC.

(a). When placed 1.16 m apart, the force each exerts on the other is 12.0 N and is *repulsive*. What is the magnitude of the charge on each?

Hint: first figure out if the two charges of the same sign or opposite.

Hint: you should end up solving a quadratic equation.

(b). For more practice, you can answer the same question as above, but given that the force is *attractive* instead.

(2). **Force between more than two point charges**: Two negative and two positive point charges (magnitude $Q = 4.15$ mC) are placed on opposite corners of a square as shown in figure 1. Determine the magnitude and direction of the force on the charge at the top-left corner due to the other three charges (for more practice, you can compute the net force on each of the other three charges also).

Hint: as usual, compute the force due to *each* of the other three charges on the top-left corner one; then add them up *vectorially*, i.e., keeping track of their magnitudes *and* directions (you can try to use the "symmetry" of the problem to simplify this process).

(3). **Electric field due to *two* point charges**: The electric field midway between two equal but *opposite* point charges is 586 N/C, and the distance between the charges is 16.0 cm. What is the magnitude of the charge on each?

Hint: will the electric fields due to the two charges add-up (or cancel)?

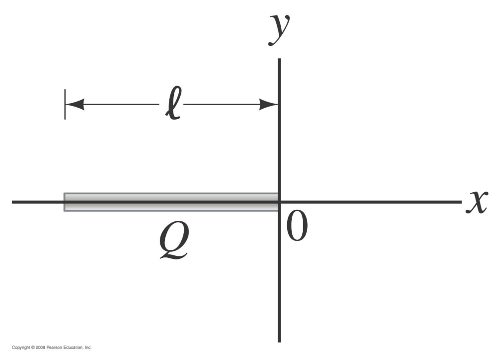

Figure 2: Problem 4

(4). **Electric field due to *line* of charge**: A thin rod of length l carries a total charge Q distributed uniformly along its length (see figure 2). Determine the electric field along the axis of the rod starting at one end – that is, find $E(x)$ for $x \geq 0$ in figure 2.

Hint: as usual, you can divide the rod into small segments (choosing a *suitable* variable denoting the position of the infinitesimal element along the rod – it might be less confusing to *not* use "x" for this, since that already denotes location of point where field is to be computed!); compute the electric field due to each such piece; then sum it up (integrate).

Note: You might wish to be careful with *sign* of this integration variable. Also, check your result in the limiting case of $x \gg l$.

(5). **Electric field due to *surface* charge**: Charge is distributed uniformly over a *large* square plane of side l, as shown in figure 3. The charge per unit area is σ. Determine the electric field at a point P a distance z above the center of the plane, *in the limit* $l \to \infty$.

Hint: Divide the plane into long narrow strips of width dy (as shown in figure 2) and use the result for the electric field due to a long line of uniform (linear) charge density λ at a distance d away from it (done in lecture and example 21-11 of Giancoli), i.e., $E = \lambda / (2\pi\epsilon_0 d)$; note that this distance, i.e., from the midpoint of each such strip to point P, will vary with the location of the strip. Then, sum the fields due to each strip (being careful with the direction of each such field) in order to get the total field at P.

Note: as usual, use symmetry of the problem in order to simplify the last step.

Note: just to repeat, the square plane is to be taken to be "infinite": you can do this at the stage of computing **E** due to each strip itself.

(6). **Gauss's law for *sphere* of charge (*no* integration required)**: An electric charge Q is distributed uniformly throughout a solid sphere of radius r_0: see figure 4. Determine the electric field (both magnitude *and* direction):

(a). outside the sphere, i.e., $r > r_0$, where r is the radial distance from the center of the sphere.

(b). inside the sphere ($r < r_0$).

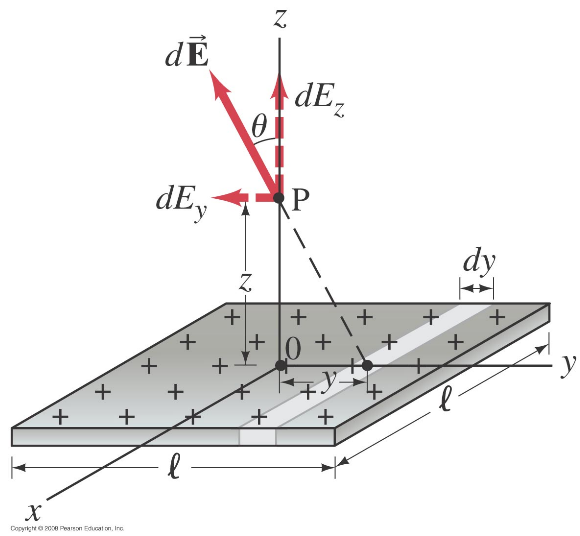

Figure 3: Problem 5

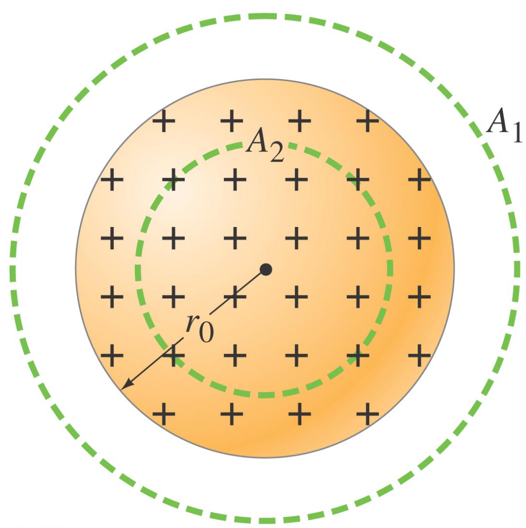

Figure 4: Problem 6 and 8

Hint: What do you expect the *direction* electric field to be for a spherical charge distribution? Your answer to this question should inform you as to what Gaussian surface to use (see figure 4).

Note: for the case (b), be careful about how much of the total charge Q is contained inside the Gaussian surface (as relevant for RHS of Gauss's law).

Note: Recall a general "strategy" for computing $\mathbf{E}$ using Gauss's law:

(1). choose the Gaussian surface appropriately, i.e., (a). needless to say, it should pass through the point P where we want $\mathbf{E}$ so that Left-hand side (LHS) of Gauss's law (electric flux) will contain the desired E and (b). conform to the "symmetry/geometry" of the charge configuration (hence the direction of the electric field itself); for example, for a long (straight) line of charge or a charged cylinder (whether solid or shell), where we expect the electric field to be directed radially outward/inward from the axis, the Gaussian surface should be a cylinder.

(2). LHS of Gauss's law then typically (but one should nonetheless check it!) reduces to simply $\sim$ (desired) E (which is due to *all* changes, whether inside or outside) $\times$ (appropriate) area *if* the electric field is along the direction of the normal to the some part the closed surface. On the other hand, on other parts of the closed surface, the electric field might be perpendicular to the normal to the surface (or electric field is tangential to the surface), in which case the relevant contribution to the flux vanishes.

(3). RHS, i.e., charge enclosed, can more subtle: again, only charge *in*side the above Gaussian surface counts, whether it is a discrete/point charge or (part of) a continuous charge distribution; in latter case, it might schematically looks like $\sim$ (appropriate) length/area/volume $\times$ right type of charge density (and if charge density is *not* a constant, then one has to start with infinitesimal version of length/area/volume, then integrate as usual).

(7). **Gauss's law for a *non*-uniformly charged solid cylinder (integration required)**: Recall that we computed electric field (using Gauss's law) for the case of a *uniformly* charged (infinitely) long solid cylinder in class. Now, consider a (infinitely) long solid cylinder of radius R with a *non*-uniform volume charge density given by $\rho = \alpha r$, where α is a constant and r is the radial distance from axis of the cylinder: see figure 5. Determine the (magnitude and direction of) electric field in the two regions:

(a). outside the cylinder (i.e., at a point with $r > R$) and

(b). inside the cylinder (i.e., $r < R$).

Do your results for these two cases "match" at the boundary of the cylinder (i.e., at $r = R$)?

Hint: Will the direction of electric field (thus choice of Gaussian surface) be similar to what we did before (i.e., uniformly charged cylinder)? For computing enclosed charge, i.e., RHS of Gauss's law, (in the case at hand) you can divide the solid/charged cylinder into *thin* cylindrical shells of radius r[1], thickness dr and length corresponding to that of the Gaussian surface/cylinder (see the cross-sectional view of the charged cylinder in figure 5)[2] and compute the charge in each shell (can the charge density be taken to be *approximately* the same throughout this *shell*?); then sum/integrate (appropriately) over such shells. As usual, for the case of determining electric field at a point inside the cylinder, will all of the charge in the length of cylinder corresponding to the Gaussian surface be enclosed by this surface?

[1]Note that r here is a "dummy" variable, i.e., it is to be integrated over, thus it is *not* to be confused with location of field point!

[2]This division is sort of analogous to that done – in lecture – for a circle (i.e., into concentric rings) in order to compute its area via an integral; see also example 21-12 from Giancoli.

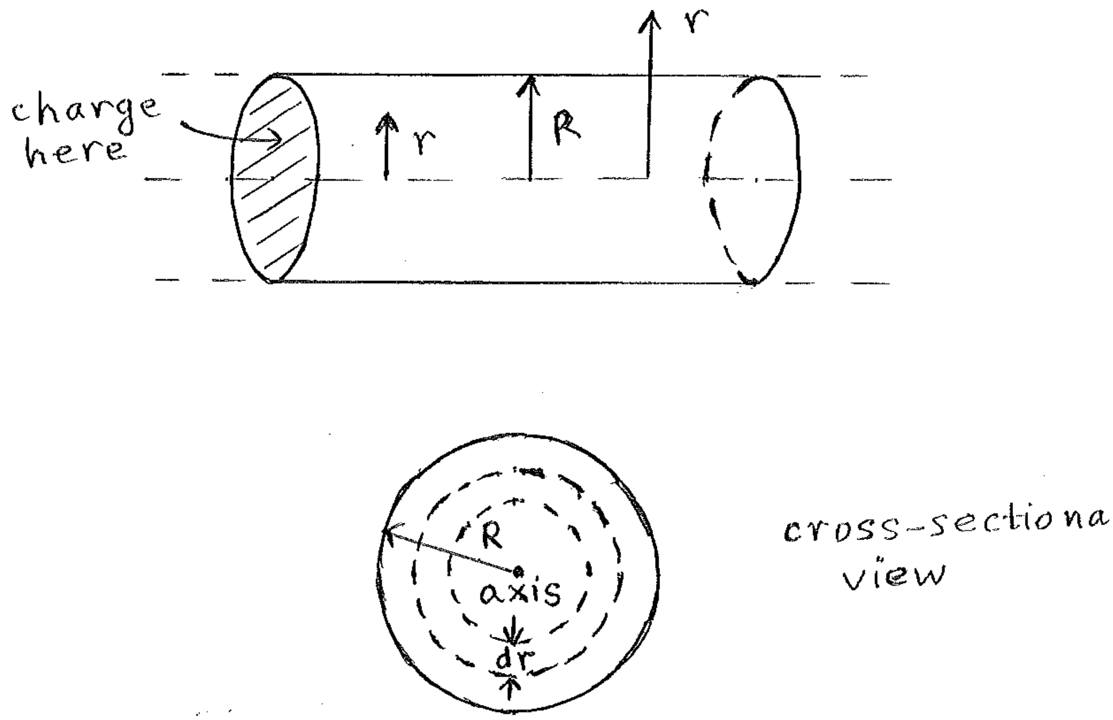

Figure 5: Problem 7

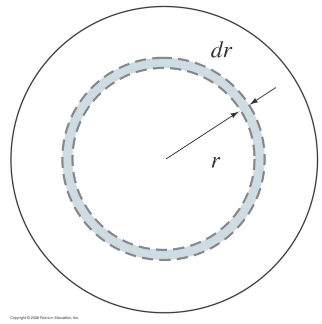

Figure 6: Problem 8

(8). **Gauss's law for a *non*-uniformly charged solid *sphere* (integration required)**: Consider the spherical version of above cylindrical problem, i.e., a solid sphere of radius r_0 as in figure 4 for problem 6 above, but now with a *non*-uniform volume charge density given by $\rho = \alpha r^2$, where α is a constant and r is the radial distance from the center of the sphere. Determine the (magnitude and direction of) electric field in the two regions:

(a). outside the sphere (i.e., at a point with $r > r_0$) and

(b). inside the sphere (i.e., $r < r_0$).

Do your results for these two cases "match" at the boundary of the sphere (i.e., at $r = R$)?

Hint: by now you know what to do! Anyway, just to repeat, choose Gaussian surface appropriately, express the electric flux through it (LHS of Gauss's law) in terms of the (wanted) electric field and compute charge enclosed by this surface (RHS). For the last step, divide the solid/charged sphere into *thin* spherical shells of radius r and thickness dr: see figure 6.

(9). **Gauss's law for solid cylinder and *thick* cylindrical shells: *no* integration required**: (This is sort of the cylindrical version of HW 4.1 – which is for a *thick* spherical shell, combined with solid cylinder done in lecture.). A very long solid cylinder of radius R_1 is uniformly charged with a charge density ρ. It is surrounded by a concentric cylindrical tube (*thick* shell) of inner radius R_2 and outer radius R_3 (as shown in figure 7) and it too carries a uniform charge density ρ. Determine the electric field as a function of distance r from the axis of the cylinders for

(a). *out*side the *entire* cylindrical charge configuration (i.e., at a point with $r > R_3$) and

(b). inside the thick cylindrical shell (i.e., $R_2 < r < R_3$).

(c). *in-between* the solid cylinder and the thick cylindrical shell (i.e., $R_1 < r < R_2$)

(d). *in*side the solid cylinder (i.e., $r < R_1$)

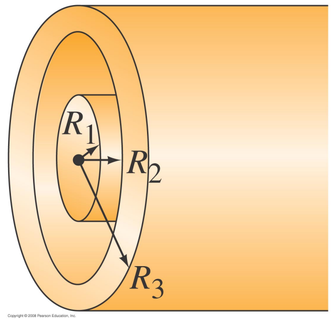

Figure 7: Problem 9

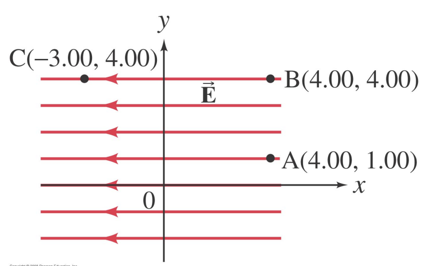

Figure 8: Problem 10

(10). **Potential difference from electric field**: A *uniform* electric field $\mathbf{E} = -4.20\ N/C\hat{\mathbf{i}}$ points in the negative x-direction as shown in figure 8. The x and y coordinates of points A, B and C (in meters) are given in the figure. Determine the *differences* in potential (a). V_{BA}, (b). V_{CB}, and (c). V_{CA}.

Hint: recall the potential difference between two points is *in*dependent of the path used to compute it in terms of electric field: so, choose the path wisely!

(11). **Potential due to *point* charges**: Two equal, but opposite charges are separated by a distance d as shown in figure 9. Determine (*directly* from the given charges) a formula for $V_{BA} = V_B - V_A$ for points A and B on the line between the charges as shown.

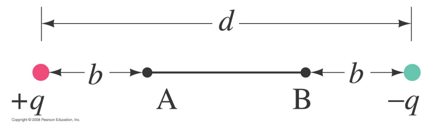

Figure 9: Problem 11

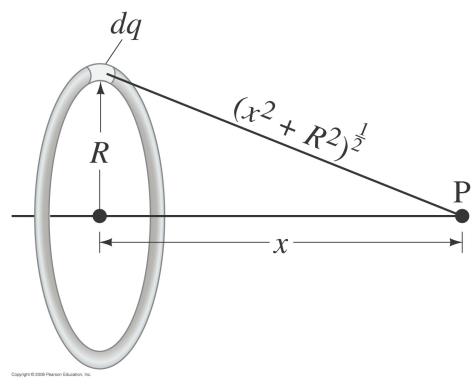

Figure 10: Problem 12

(12). **Potential (and electric field) due to a charge *distribution***: A thin circular ring of radius R has charge $+Q/2$ uniformly distributed on the top half, and $-Q/2$ on the bottom half (see figure 10): again, charge is *opposite* in sign on the two halves of the ring.

(a). Calculate the electric potential at a point a distance x along the axis through the center of the ring (see figure 6) *directly* from the charge distribution. (**Hint**: think about possible *cancellation* between different parts of the ring.)

(b). Simply based on your result above for the potential, what can you say about the x-component of the electric field at a distance x along the axis?

(c). Similarly (i.e., based *only* on the above result on potential along the x-axis), can you figure out (or obtain *any* information regarding) the y (or z) component of the electric field? Explain your answer.

(d). If answer to part (c) is "No", then use the "standard" method for this purpose, i.e., determining electric field from charge distribution by superposition principle, (once again, make use of possible cancellations between effects from different parts of the ring).

[Compare the above to the case of charge $+Q$ uniformly distributed over the *entire* ring in examples 21-9, 23-8 and 23-11 (a) of Giancoli.]

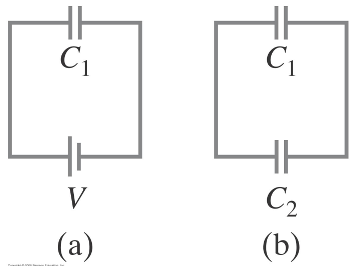

Figure 11: Problem 14

(13). Four equal point charges, Q, are fixed at the corners of a square of size b.

(a). What is their total electrostatic potential energy?

(b). How much potential energy will a fifth charge, Q, have at the *center* of the square (relative to $V = 0$ at infinity)?

(14). **Basics of capacitance**: A 7.7-μF capacitor is charged by a 125-V battery [see figure 11 (a)] and then *dic*connected from the battery. When this capacitor (C_1) is then connected [see figure 11 (b)] to a second (initially *un*charged) capacitor, C_2, the final voltage on each capacitor is 15 V. What is the value of C_2?

Hint: First determine the initial charge on the 7.7-μF capacitor; then use the fact that while charge can be re-distributed, the *total* charge is conserved in the second stage.

Note: it might be useful to solve this problem "symbolically" first (say, denoting the initial and final voltages as V and V' (respectively) and the two capacitances as $C_{1,2}$ (as shown in figure); then plug in the numbers.

Note: This problem is similar to HW 5.2, example 24-7 of Giancoli and problem 24-6 of Ginacoli, which was done in lecture.

(15). **Two capacitors in Series and Parallel**: Two capacitors connected in parallel produce an equivalent capacitance of 8 μF. However, when connected in series, the equivalent capacitance is only 1.5 μF. What is the individual capacitance of each capacitor?

Hint: you should end up solving a quadratic equation.

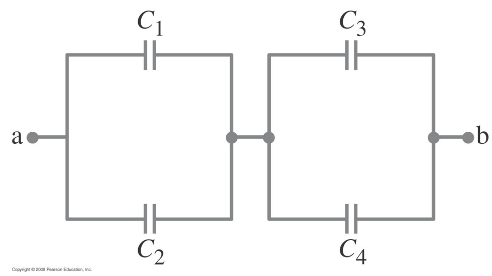

Figure 12: Problem 16

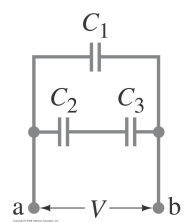

Figure 13: Problem 17

(16). **More than two capacitors in Series and Parallel (I)**: (a). Determine the equivalent capacitance between points a and b for the combination of capacitors shown in figure 12.

[**Note**: if you wish to begin with a simpler case, then choose *all* capacitances to have the *same* value.]

(b). Determine the charge on and the voltage across each capacitor if $V_{\mathrm{ba}} = V$.

Note: part (a) of such problems (see also HW 5.4 or the one done in lecture, which was based on example 24-5 of Giancoli) ask for equivalent capacitance of a large network, which is to be obtained by multiple uses of the series and parallel formulae for a pair of capacitors.

Part (b) then is about charge/voltage for each capacitor of the original network; for obtaining this, one typically has to "work backward" through the analysis done for part (a): see HW 5.4 or lecture notes or example 24-6 from Giancoli.

(17). **More than two capacitors in Series and Parallel (II)**: For the combination of capacitors shown in figure 13, determine

(a). the equivalent capacitance between points a and b assuming $C_1 = C_2 = C_3 = 22.6\mu$F;

(b). the charge on and the voltage across each capacitor for $V = 10.0$ V and

(c). the (electric) *energy* stored in each capacitor, thus the total energy stored in the capacitor network

[**Note**: you might wish to *first* solve this problem "symbolically" (i.e., just using C for the common capacitance value etc.), then plug-in numerical values.

(18). **Change in value of capacitance (and hence voltage across it) up on filling it with dielectric**: Two identical capacitors are connected in parallel and each acquires a charge Q_0, when connected to a source of voltage V_0. The voltage source is then *dis*connected and a *dielectric* ($K = 3.2$) is inserted to fill the space between the pates of *one* of the capacitors. Determine the charge now on (hence voltage across) each capacitor.

Hint: what happens to the capacitance value when the dielectric is inserted?

Note: Just like in HW 5.2, problem # 14 above, example 24-7 of Giancoli and problem 24-6 of Ginacoli (which was done in lecture), use the fact that while charge can be re-distributed, the total charge is conserved (and that the potential difference between two points is independent of path along which we evaluate it, with connecting/conducting wires being equipotentials, i.e., there is no potential "drop" across them).

(19). **Basics of resistance of a wire**: Two wires made of the same material have the same resistance. If one has twice the length of the other, what is the *ratio* of the diameter of the longer wire to the diameter of the shorter wire?

(20). **Basics of power and Ohm's law**: How many 75-W lightbulbs, all connected to a 120 V line (i.e., in *parallel*), can be used with*out* blowing a 15-A fuse?

(**Hint**: you might wish to first calculate the current drawn by *each* bulb, then determine the total current: can the latter value exceed the rating of the fuse?)

(21). **Microscopic understanding of current**: A 0.65-mm-*diameter* copper wire (of resistivity 1.68×10^{-8} Ω.m and free electron number density 8.5×10^{28} m^{-3}) carries a tiny current of 2.3 μA. Estimate

(a). the electron drift velocity

(b). the current density and

(c). the electric field in the wire.

(22). **Power delivered to resistances and their parallel combination**: A 75-W, 110-V bulb is connected in *parallel* with a 25-W, 110-V bulb. What is the *net* resistance? (**Hint**: you might wish to first calculate the resistance of each bulb.)

(23). **Series and parallel combinations of resistances**: Three resistors with the *same* resistance value R can be connected together in *four* different way, making combinations of series and/or parallel circuits. What are these four ways and what is the net resistance in each case?

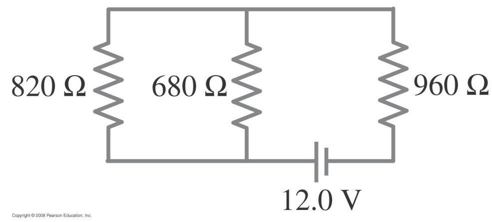

Figure 14: Problem 25

(24). ***Non*-zero resistance for conducting wire**: Eight bulbs are connected in *parallel* to a 110-V source by two long wires of *total* resistance 1.4Ω. If a current of 240 mA flows through *each* bulb, what is the resistance of each, and what *fraction* of the total power is wasted in the *wires*?

Hint: it might be useful to *draw* a picture of this circuit.

Hint: *Un*like what is assumed usually, will there be a voltage drop across the connecting wires in *this* case (so that will the voltage across each bulb be the *full* 110-V of the source)?

(25). **Network of resistances**: For the circuit shown in figure 14, determine

(a) the equivalent resistance and

(b) the voltage across each resistor.

Note: 1st part of such a problem (see also HW 7.3 or the one done in lecture, which was based on examples 26-4, 26-5 of Giancoli) ask for equivalent resistance of a large network, which is to be obtained by multiple uses of the series and parallel formulae for a pair of resistors.

2nd part then is about current through/voltage across for each resistor of the original network; for obtaining this, one typically has to "work backward" through the analysis done for 1st part: see lecture/above homework problem or examples from Giancoli.

(26). **Another network of resistances**: Consider the circuit connected to the battery in figure 15.

(a). What is its net resistance?

(b). What is the current through each resistor?

(c). What is the potential difference between points A and B?

(27). **Power delivered to series and parallel combinations of resistances**: A voltage V is applied to n *identical* resistors connected in *parallel*. If the resistors are instead all connected in series with the *same* applied voltage, by what factor does the *total* power delivered to the resistors change?

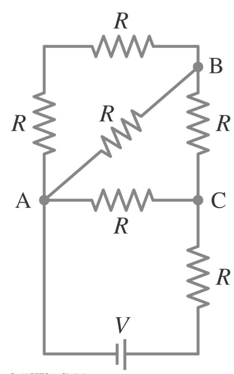

Figure 15: Problem 26

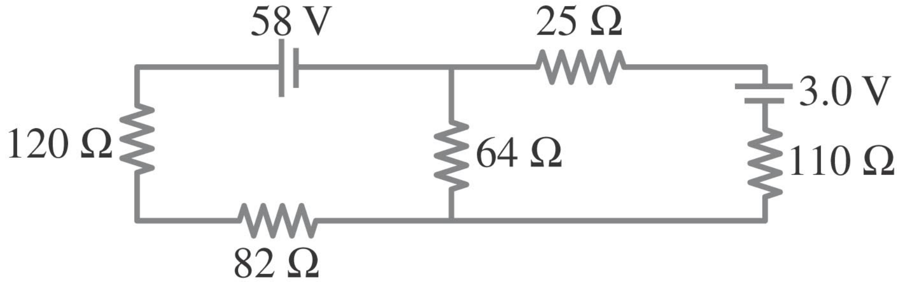

Figure 16: Problem 28

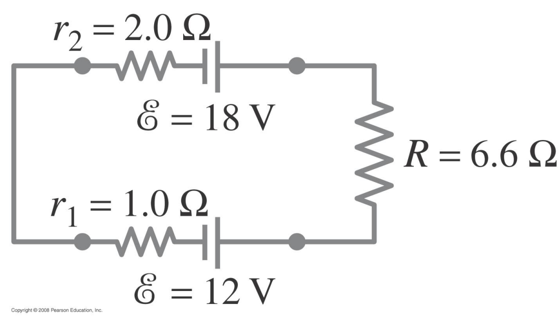

Figure 17: Problem 29

(28). **Kirchhoff's rules for circuit with two loops**: Calculate the currents in each resistor of above figure.

Recall a general "recipe/strategy" for solving for currents etc. in a circuit using Kirchhoff's rules:

(1). Label the *current* in each separate branch of the circuit [i.e., in-between two junctions (where connecting wires meet)], choosing a *direction* for it: typically, these are the unknowns in the problem.

(2). Apply Kirchhoff's *junction/current* rule at one or more junctions of the circuit, i.e., sum of currents entering is equal to sum leaving.

(3). Apply Kirchhoff's *voltage/loop* rule to one or more loops of the circuit (again, choose a *direction* for going around each loop): sum of voltages (with appropriate signs) across each component around the loop is zero. In particular,

(a). For a resistor with resistance R and current I flowing through it, the voltage is taken to be $-RI$ if the chosen direction of loop [from step (2) above] is *same* as that (assumed) of the current [from step (1) above] (and $+RI$ if the two directions are opposite).

(b). For a battery, the (usually given) voltage is taken with a positive sign if the chosen direction of the loop is from negative to positive terminal, whereas it is negative if the direction of the loop is the opposite.

(4). Check that there are as many equations as unknowns: again, voltages across resistances can be written in terms of currents, so do not count as further unknowns. If you are "short" of equations, then you need to apply Kirchhoff's rules to additional junctions or loops.

(5). Solve algebraically (the linear, *in*homogeneous system of equations) for the unknown (currents): if value of a current turns out to be negative, then it just implies that the actual direction is opposite to what was assumed to start with. You can check the answers by using any "extra" loop or junction rule equations.

(29). **Kirchhoff's rules for circuit with single loop/battery internal resistances**: Determine the terminal voltage of each battery in the above figure (as usual, r denotes internal resistance).

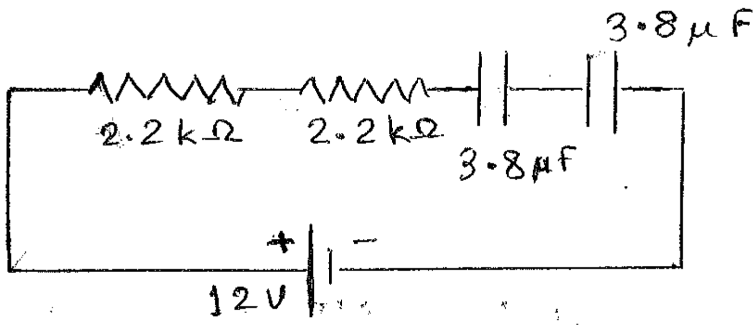

Figure 18: Problem 30

(30). **RC circuit**: Two 3.8-μF capacitors, two 2.2-kΩ resistors, and a 12.0-V source are connected in *series*: see figure.
(a). What is the equivalent resistance of the circuit?
(b). What is the equivalent capacitance of the circuit?
(c). What is the effective time constant of the circuit?
(d). What is the initial current in the circuit (i.e., when the capacitors are *un*charged)?

Starting from the above state (again, with no charge on capacitors, but current flowing in the circuit), how long does it take
(e). for the *current* to drop from its initial value to 1.50 mA?
(f). for the *energy* stored in each of the capacitors to reach 75% of its maximum value?

(31). **Motion of charged particle in a magnetic field**: For a particle of mass m and charge q moving in a circular path of radius r in a magnetic field B (see figure), show that
(a). its kinetic energy is proportional to r^2, i.e., the square of radius of curvature of its path and
(b). its angular momentum is $L = qBr^2$, about the center of the circle.

Note: the idea here is to *re*-write kinetic energy and angular momentum in terms of the "given" quantities, i.e., q, m, B and r.

(32). **Force due to magnetic field on a current**: A stiff wire of length l is bent at a right angle in the middle. One section lines along the z axis and the other is along the line $y = x$ in the xy plane. A current I flows in the wire – down the z axis and out the line in the xy plane. The wire passes through a *uniform* magnetic field of magnitude B along the x direction: see figure. Determine
(a). the force on the half of the wire along the z axis,
(b). the force on the half in the xy plane and

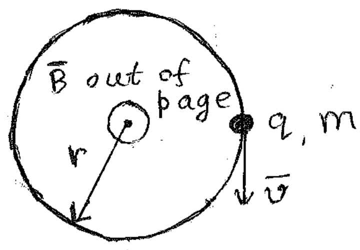

Figure 19: Problem 31

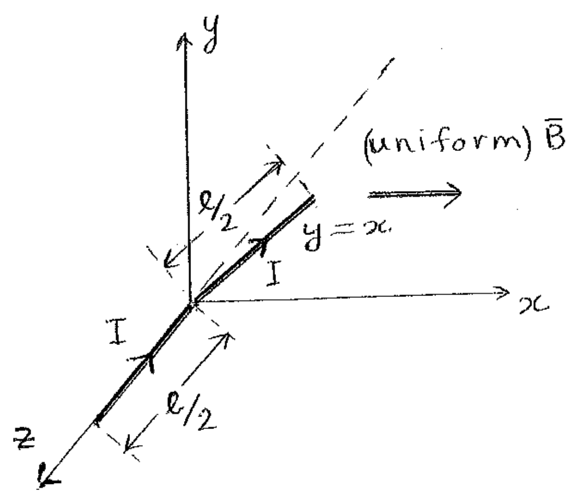

Figure 20: Problem 32

(c). the total force on the wire.

(33). **Magnetic field due to *two* currents**: Let two long parallel wires, a distance d apart, carry equal currents I in the same direction. One wire is at $x = 0$, the other at $x = d$: see figure. Determine the magnetic field (magnitude and direction) along the x axis between the wires as a function of x

(a). due to the wire on the left

(b). due to the wire on the right

(c). due to the two wires combined.

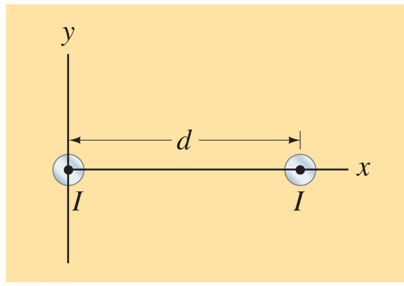

Figure 21: Problem 33

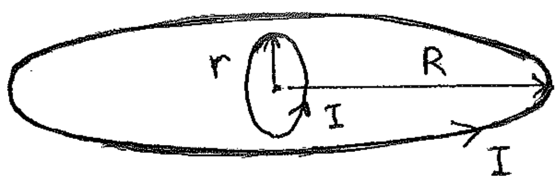

Figure 22: Problem 34

(34). **Torque on a current loop due to another current**: A small loop of wire of radius r is placed at the center of a (much) larger wire loop with radius R. The planes of the two loops are *perpendicular* to each other, and a current I flows in each: see figure.

(a). What is the magnitude and direction of the magnetic field due to the current in the larger loop at its center?

(b). What is the magnetic dipole moment of the smaller loop?

(c). Using the above results and *assuming* $r \ll R$, estimate the torque the large loop exerts on the smaller one. (**Hint**: can you take the magnetic field due to the larger loop over the area of the smaller loop to be *approximately* uniform?)

(35). **Ampere's law**: A long (thin) straight wire carries a current I, while a concentric cylindrical (thin) shell of radius R carries *twice* the current (i.e., $2I$), but in the *opposite* direction: see figure. Using Ampere's law, determine the magnitude *and* direction of magnetic field for the two cases below.

(a). *in*side the shell, i.e., at a radial distance from the wire of $r < R$ (**Hint**: will the current in the *outer* shell contribute to the RHS of Ampere's law in this case?)

(b). outside the shell (i.e., at $r > R$). (**Hint**: will the current in the inner wire and the outer shell contribute with the same or opposite *sign* to RHS of Ampere's law?)

Hint: you might find it useful to study Example 28-6 (also done in lecture), Example 28-7 (also done in lecture) and its variation: HW 10.1, which is Problem 31 of chapter 28.

Note: Recall a general "strategy" for computing the magnetic field ($\mathbf{B}$) using Ampere's law (also given at bottom of page 740 of Giancoli):

(1). Based on the symmetry of the current configuration, make an "educated guess" for the direction of $\mathbf{B}$; for example, for an "axial" current (whether along a long, straight wire or a long, thin cylindrical shell), we expect $\mathbf{B}$ to be "circumferential", i.e., $\mathbf{B}$ lines form concentric circles, with *magnitude* of $\mathbf{B}$ being the same on each such circle.

You can simply assume one "sign" of $\mathbf{B}$, for example, say, clockwise in above situation: if (in the end) the (magnitude of) B that you compute turns out to be *negative*, then $\mathbf{B}$ is actually *counter*-clockwise.

(2). Choose the Amperian loop [i.e., *closed* path, along which (line) integral of $\mathbf{B}$ on the Left-hand side (LHS) of Ampere's law is computed] appropriately, i.e., (a). needless to say, it should pass through the point P where we want $\mathbf{B}$ so that the LHS of Ampere's law will contain the desired B and (b). conform to the "symmetry/geometry" of the charge configuration etc.; for example, for an axial current mentioned above, the Amperian loop should be a concentric circle.

Also, (c). it is "convenient" to choose the *direction* of going around the Amperian loop to be the "same" as that of the assumed $\mathbf{B}$. Just to be clear, it is also Ok to make the opposite choice of going around the loop: in this way, there might be an "extra" negative sign in the step below (computing LHS of Ampere's law), of course giving the same final answer (once you are careful in step 4 below about sign of the enclosed current).

(3). LHS of Ampere's law then typically (but one should nonetheless check it!) reduces to simply $\sim$ (desired) B (which is due to *all* currents, whether inside or outside) $\times$ length of appropriate part of the Amperian loop, i.e., there could be other pieces of the Amperian loop which do not give any contribution due to either B vanishing or $\mathbf{B} \perp$ tangent to the path.

(4). RHS, i.e., current enclosed, can be more subtle: again, only current *piercing/threading through* the surface whose boundary is the Amperian loop on LHS counts, whether it is a discrete/line current or (part of) a continuous current distribution; in the latter case, the enclosed current schematically looks like $\sim$ area$\times$ current density (i.e., current per unit area) and if current density is *not* a constant, then one has to start with infinitesimal version of area, then integrate as usual.

Be careful with the *sign* of each part of the (enclosed) current, i.e., follow the *right-hand rule*: if the fingers of your right hand indicate the direction of integration around the closed path, then your thumb defines the direction of *positive* current.

(36). **Faraday's law**: A single circular loop of wire is placed inside a long solenoid with its plane perpendicular to the axis of the solenoid: see figure. The area of the loop is A_1 and that of the solenoid, which has n turns per unit length, is A_2. A current of $I = I_0 \cos \omega t$ flows in the solenoid turns.

(a). What is the magnetic field inside the solenoid?

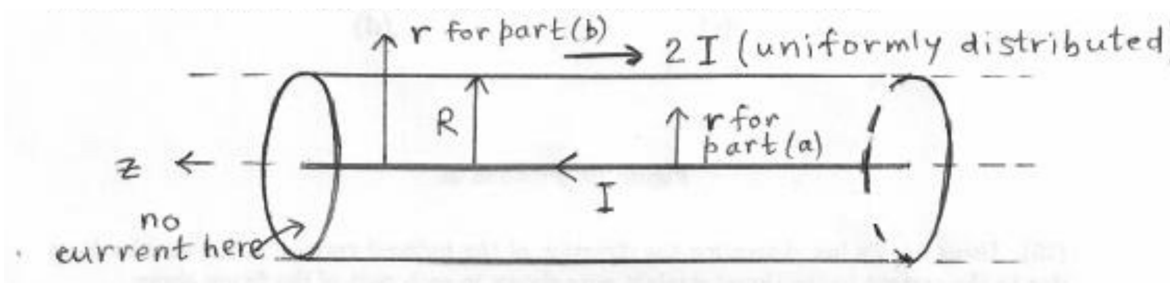

Figure 23: Problem 35

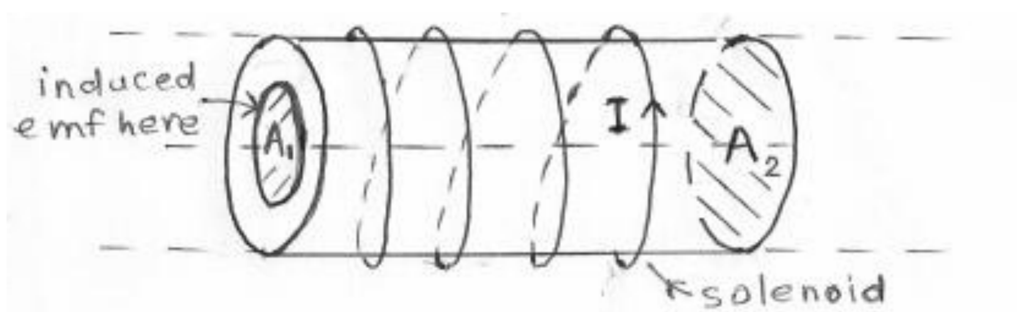

Figure 24: Problem 36

(b). Based on above result, what is the magnetic flux through the smaller loop?
(c). What is then the induced emf in the small loop?

(37). **Lenz's law**: Using Lenz's law determine the direction of the induced current in the circular loop due to the current in the (long) straight wire shown in each part of the figure above. (In addition to notes below, you might find it useful to look at HW 10.3, i.e., problem 8 of chapter 29 from Giancoli, and Example 29-4 from Giancoli.)

Note: Recall that Lenz's law states the induced current flows in a direction such that the magnetic field that it (i.e., the induced current) creates *opposes* the original *change* in magnetic flux (which could be due to *another* current changing or a permanent magnet moving etc.) that induced the current in the first place.

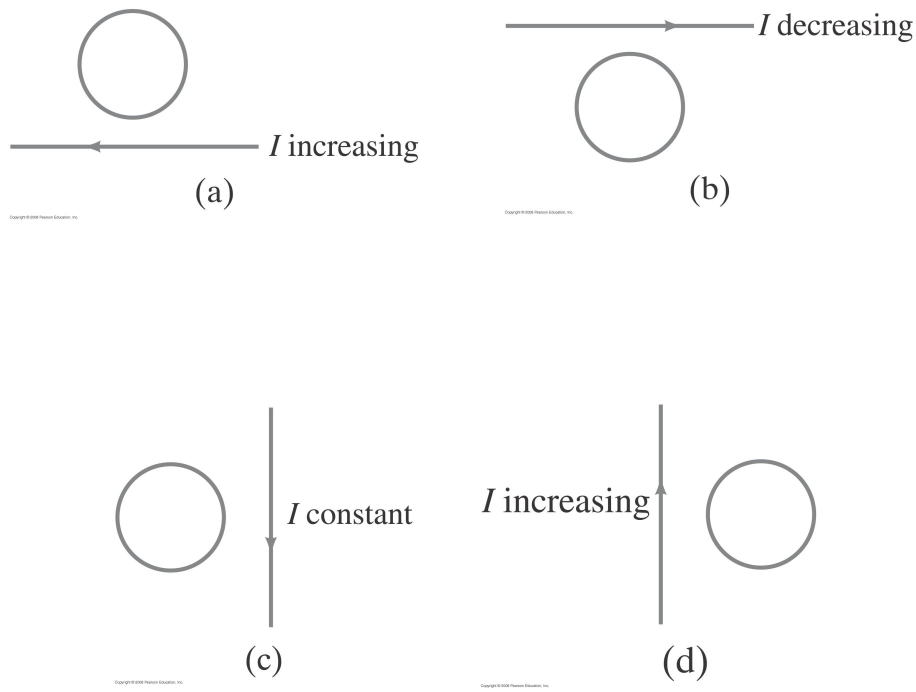

Figure 25: Problem 37

Hence, we have the general *strategy* given below for using Lenz's law in order to figure out the *direction* of the induced current, which is also outlined at the top of page 763 of Giancoli (and it was also done in lecture): Example 29-4 from Giancoli or problem # 37 from Wednesday session will also be useful here.

(1). Figure out whether the magnetic flux (due to *another* current or a permanent magnet) inside the loop is decreasing, or increasing, or unchanged.

(2). The *magnetic field* due to the induced current: (a). points in the *same* direction as the external field if the inducing flux is *decreasing*; (b). points in the opposite direction from the external field if the flux is increasing; or (c). is zero if the flux is not changing.

(3). Knowing the direction of the magnetic field due to the induced current, use the right-hand rule to determine the direction of the induced *current*.

So, in *this* problem, you will first have to figure out the direction of magnetic field due to the (long) straight wire at the location of the circular loop and eventually relate the direction of current in the circular loop with that of the magnetic field that *it* creates (as per Lenz's law).

(38). **More Lenz's law**: If the solenoid shown in the figure above is being pulled away from the loop, in what direction is the induced current in the loop? Similarly, what if the solenoid is being moved *toward* the loop? (See above problem for the general strategy for such problems.)

(39). ***Two* ways to compute emf (Faraday's law *and* motional emf)**: Part of a single rectangular loop of wire (with dimensions as shown in the figure, i.e., width $w = 0.35$ m and

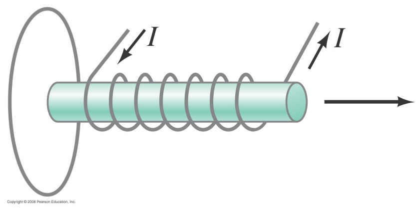

Figure 26: Problem 38

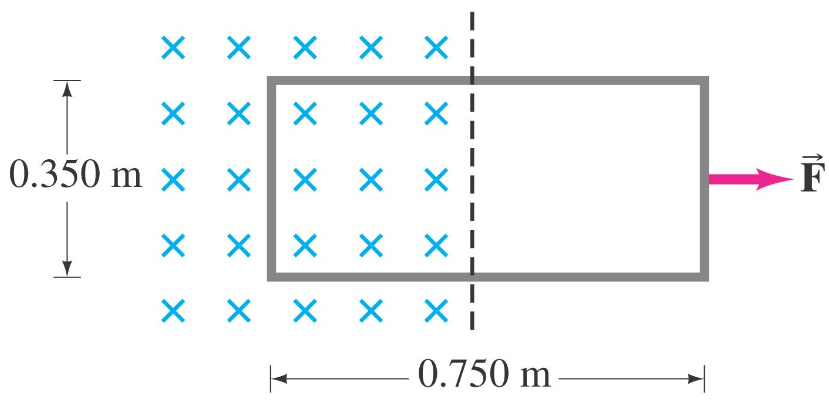

Figure 27: Problem 39

length $l = 0.75$ m) is situated inside a region of uniform magnetic field $B = 0.650$ T which is directed into the page. The total resistance of the loop is $R = 0.280\ \Omega$. Calculate the force F required to pull the loop from the field (to the right) with a constant velocity of $v = 3.4$ m/s. Neglect gravity.

Solve each of the two parts of this problem in two ways (checking that they give the same result) as follows [you might take a look at Examples 29-5 and 29-8 from Giancoli and HW 10.5, i.e., problem 31 of chapter 29 of Giancoli (versions of which were also done in lecture)]:

(a). First of all, there are two ways to compute the emf induced in the loop, thus the induced current. Namely, (i) use Faraday's law, i.e., emf is given by the rate of change of magnetic flux through the loop. Equivalently, (ii) calculate the *force* exerted by the magnetic field on the *charge carriers* (say, free electrons) as they move with velocity v to the right (as part of the loop): the induced emf is then simply the work done *per unit charge* by this (magnetic) force.

(b). Once you have determined the (induced) current in the loop, there are two ways to

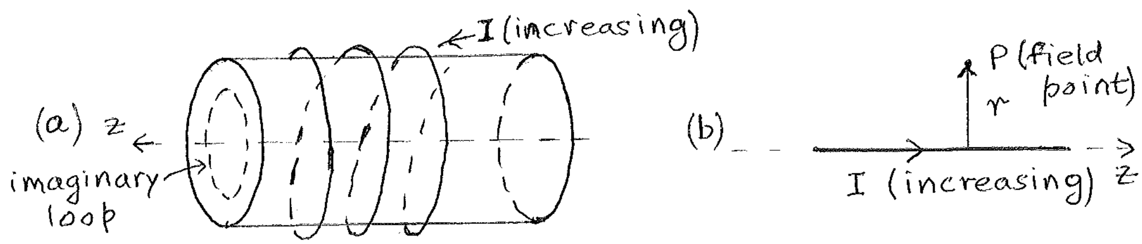

Figure 28: Problem 40

figure out the external force needed to pull the loop out of the magnetic field, i.e., (i) find the power dissipated in the resistance of the loop; this energy must then be supplied by the external force on the loop. Equivalently, (ii) you can simply compute the force due to the magnetic field on the (induced) *current*: the external force must then balance this magnetic force.

(40). **E from changing B**: This problem is about determining the *direction* of the electric field induced by a *changing* magnetic field (first case was actually done in lecture on April 23 and is also the topic of Example 29-14 of of Giancoli and HW 11.2, i.e., problem 54 of chapter 29). Recall that the induced electric field is what results in a current *assuming* there is a conducting path (i.e., a loop of wire placed etc.) at that location. However, the electric field exists even in the absence of such an "opportunity" for a current.

The first step is to obtain the direction of the magnetic field due to the current. Then, try to (carefully) use the "analogy" of Faraday's law, i.e., $\oint \mathbf{E}.d\mathbf{l} = -\frac{d\Phi_B}{dt}$, with Ampere's law, i.e., $\oint \mathbf{B}.d\mathbf{l} = \mu_0 I_{\text{encl}}$: to state the obvious, we see that $\mathbf{E}$ in former is like $\mathbf{B}$ in the latter, while Φ_B (rather its rate of change) is similar to the current threading through the surface whose boundary is the closed loop on the LHS.
[Recall (from discussion of Ampere's law) that an "axial" current (as in a thin/thick long straight wire or cylindrical shell) produces a "circumferential" magnetic field, while a circumferential current (like in a solenoid) has an axial magnetic field.]

(a). Suppose the current in a solenoid is increasing with time (see figure on left below). What is the general direction (i.e., along its axis or radial or circumferential) of the induced *electric* field inside and outside the solenoid?

If the electric field is circumferential, then is it clockwise or anti-clockwise (say, as seen from the left)? For this purpose, you can either use Faraday's law as above, being careful with various "signs" in it. Alternatively, use Lenz's law, i.e., *imagine* a conducting loop (appropriately oriented) inside/outside the solenoid: figure out the direction of current induced in it (again using Lenz's law), thus deducing that of the electric field as well. (Again, the electric field is present even if there is no such loop *in reality*.)

(b). A long, straight wire carries a current which is increasing with time (see figure on right below). What is the general direction (again, axial, circumferential or radial) of the induced *electric* field outside it?

(41). ***Mutual* inductance**: A long solenoid of radius r_1 with n_1 turns per unit length is

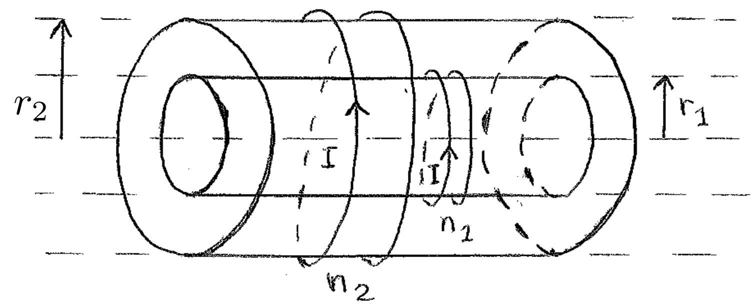

Figure 29: Problem 41

placed inside another of radius r_2 ($> r_1$) with n_2 turns per unit length, each carrying current I (see figure).

(a). What is the magnetic field inside the larger solenoid due to the current in it?

(b). Using the above result, what is the magnetic flux through the smaller solenoid (per unit length) due to the current in the larger solenoid?

(c). Determine the mutual inductance (per unit length) between the two solenoids. Does it depend on the current I? Also, compare this result to that of Example 30-1 of Giancoli (also done in lecture) and HW 11.3, i.e., problem 30-3 from Giancoli.

(42). **Magnetic energy stored (integration required)**: A long straight *thick* wire of radius R carries current I which is uniformly distributed across its cross-sectional area (see figure).

(a). Using Ampere's law (being careful: in particular, about the enclosed current!), determine the magnetic field at a point *in*side the wire a radial distance r ($< R$) from axis of the wire. (This was done in lecture and is Example 28-6 from Giancoli.)

(b). Find the magnetic energy stored per unit length in the interior of this wire. [**Hint**: is the magnetic field uniform inside the wire? If not, then divide-up the wire into infinitesimal cylindrical shells such that magnetic field *is* (approximately) a constant inside each shell, compute the magnetic energy in each shell and then integrate: see figure below. (A similar approach is to be used for HW 11.5, i.e., problem 30-20 of Giancoli.)]

(43). **LR circuit**: For the simple LR circuit shown in figure above (where the switch connecting the battery is closed at $t = 0$), determine

(a). the energy stored in the inductor L as a function of time and

(b). after how many time constants does the energy stored in the inductor reach 99.9% of its maximum value.

(44). **LC circuit**: (This problem is similar to Example 30-7 of Giancoli.) A 425-pF capacitor is charged to 135 V and then quickly connected to a 175-mH inductor: see figure above. Determine

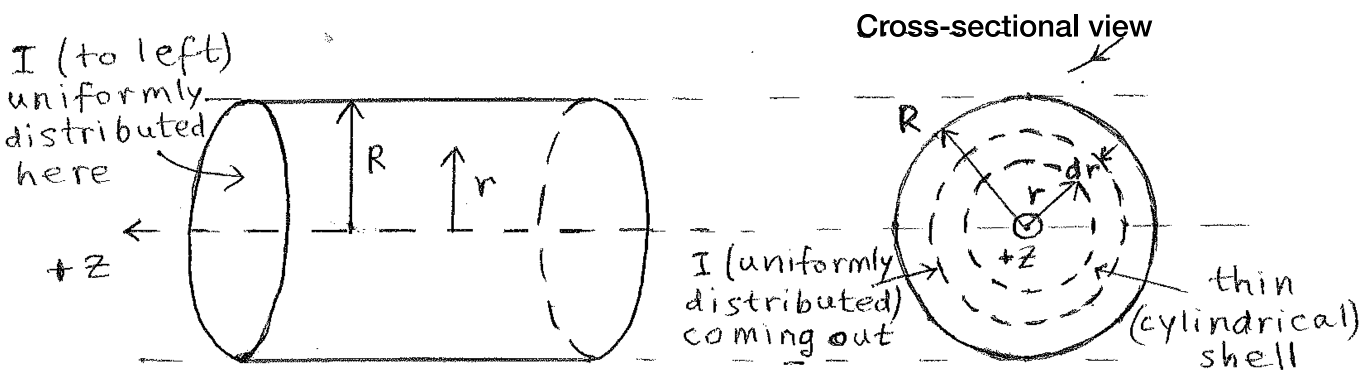

Figure 30: Problem 42

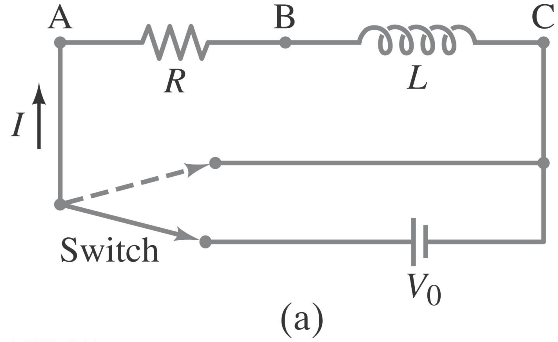

Figure 31: Problem 43

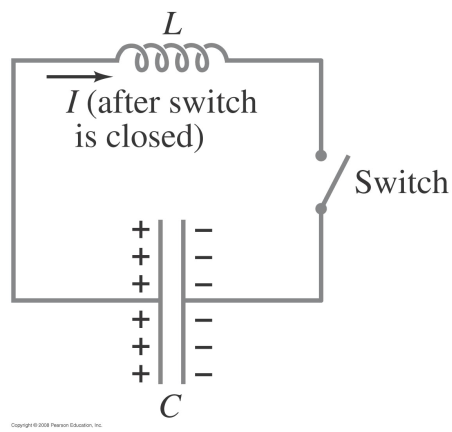

Figure 32: Problem 44

(a). frequency of oscillation,
(b). the peak value of the current, and
(c). the maximum energy stored in the magnetic field of the inductor.

(45). **Modification of Ampere's law by Maxwell**: Show that the displacement current through a parallel-plate capacitor can be written $I_D = CdV/dt$, where V is the voltage across the capacitor at any instant. (**Hint**: first obtain the electric flux through the capacitor.)

(46). **EM wave**: (This problem is another version of HW 12.5, i.e., problem 31-23 of Giancoli.) A high-energy pulsed laser emits a 1.0-ns-long pulse of average power 1.8×10^{11} W. The beam is 2.2×10^{-3} m in radius. Determine
(a). the energy delivered in each pulse
(b). the rms value of the electric field
(c). the rms value of the magnetic field